

**The impact of failure, repair and joint imbalance of  
processing time means  
buffer sizes on the performance of unpaced production  
lines**

Sabry Shaaban, Tom Mcnamara, Sarah Hudson

► **To cite this version:**

Sabry Shaaban, Tom Mcnamara, Sarah Hudson. The impact of failure, repair and joint imbalance of processing time means

buffer sizes on the performance of unpaced production lines. International Journal of Manufacturing Technology and Management, Inderscience, 2015. <hal-01205567>

**HAL Id: hal-01205567**

**<https://hal-esc-rennes.archives-ouvertes.fr/hal-01205567>**

Submitted on 29 Sep 2015

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# The impact of failure, repair and joint imbalance of processing time means & buffer sizes on the performance of unpaced production lines

Sabry Shaaban

Department of Economics, Strategy and Organization  
Groupe ESC La Rochelle - Cerege  
102 Rue de Coureilles  
17024 La Rochelle, France  
Email: [shaabans@esc-larochelle.fr](mailto:shaabans@esc-larochelle.fr)

Tom McNamara

Sarah Hudson

Department of Finance and Operations  
ESC Rennes School of Business  
2 Rue Robert d'Arbrissel  
35065 Rennes, France  
Email: [tom.mcnamara@esc-rennes.fr](mailto:tom.mcnamara@esc-rennes.fr)  
Email: [sarah.hudson@esc-rennes.fr](mailto:sarah.hudson@esc-rennes.fr)

**Abstract:** This article studies the performance of unpaced serial production lines that are subject to breakdown and are imbalanced in terms of both of their processing time means (MTs) and buffer storage capacities (BCs). Simulation results show that the best pattern in terms of throughput is a balanced line. With respect to average buffer level; the best configuration is a monotone decreasing MT order, together with an ascending BC arrangement. Statistical analysis shows that BC, combined patterns of MT and BC imbalance, line length and degree of imbalance all contribute significantly to performance.

**Keywords:** unreliable unpaced serial lines; simulation; unequal mean operation times; uneven buffer capacities; patterns of imbalance; throughput; average buffer level.

## 1. Introduction

Studies of asynchronous unpaced serial production lines are often concerned with achieving balance. However, while a balanced allocation strategy, where all workstations have equal mean processing times and all buffer stores are provided with an even distribution of capacity is the simplest, it may not be possible in real-world systems where breakdown, delayed arrival of raw material and processing time variability interrupt the ideal functioning of a balanced line. In a review of the research into line balancing, Tempelemeier (2003) noted that real world systems do not have stations with identical mean processing times, so we can propose that the investigation of unbalanced lines could be of practical use in understanding how best to deal with this fact.

A number of publications on the subject of reliable unbalanced lines in recent years have shown that imbalance in terms of operating times means (MT), coefficients of variation (CV) and buffer capacity (BC) allocation does not necessarily lead to a deterioration in performance when compared to a balanced line, and in some cases can outperform a balanced configuration (Shaaban, 2011; Shaaban and McNamara, 2009; and Shaaban and Hudson, 2009). Whether this is also true in the case of lines suffering breakdown has not been much investigated, and it is here that this paper hopes to contribute to understanding the factors which need to be considered and prioritized when dealing with unreliable unbalanced lines.

In this paper we present the results of simulations of unpaced serial production lines where buffer capacity is unevenly allocated between buffer stores (BC imbalance), with differing mean station times (MT imbalance) in different configurations. We look at the effects of combined MT and BC imbalance on throughput and average buffer level. Two line lengths ( $N = 5$  and  $N = 8$ ) are investigated, and two cases are considered where total buffer availability is low or high. In addition, three degrees of MT imbalance are studied, where the differences between slower and faster stations are low (2%), medium (5%) and high (12%). We use statistical analysis to come to some conclusions about how each of these factors affects performance separately and in combination, to provide some insights for managers on those factors that impact performance the most.

This paper is organized as follows. First, the relevant literature is reviewed. Next, the objectives and research questions of the study are presented. Subsequent sections discuss the methodology and experimental design and provide the simulation results and analysis. The last part provides discussion of the results and research implications.

## 2. Literature Review

For the purposes of this article, we will mainly concentrate on the work closest to the particular theme of this paper, the effects of combined MT and BC imbalance in unreliable lines.

Several optimization studies have been carried out on unreliable lines with unequal station mean times. The aim of these is to address the buffer allocation problem, i.e. trying to generate algorithms to position buffers in order to achieve optimum output. An algorithm for the optimal allocation of buffers in transfer lines subject to breakdowns was developed by Ho et al. (1979). The distinguishing feature of their method is that it calculates the reactivity, or degree to which an increase in a unit of buffer increases the output of a line. The algorithm was applied to a 7-station unbalanced line. Ho et al. did not specify the pattern that an optimal

allocation of buffer would take. Gershwin and Schor (2000) presented algorithms for the efficient placement of buffers in production lines. Results were compared to those of Ho et al. (1979) and found to be efficient. Nahas et al. (2006) developed a predictive method for the efficient allocation of buffers that incorporated a degrading ceiling approach. The authors looked at a seven-station serial production line having a total buffer capacity of 54 units, utilizing the same MTs as those used by Ho et al. (1979) and found that their search method to be efficient.

Altiok and Stidham (1983) addressed the same issue with the dependent variable being profits rather than output. They developed an algorithm for the optimal allocation of buffers with regards to profits, taking into account the item holding cost, variable production cost and revenue per item. The authors studied a three-station, exponentially distributed MT unbalanced line, in which each station was subject to machine failure. The MTs were arranged in an inverted bowl fashion. They found it advantageous to assign lower BC before the bottleneck station than after it, due to the high holding costs involved and the fact that buffers would build up as a result of the first station being 25% faster than the middle one.

Heavey et al. (1993) developed a solution method for the prediction of TR. Stations with different average service times that were exponentially or Erlang distributed and varying buffer capacities were studied. The lines are subject to failure, with failure and repair rates taking on the same distributions as the service times. The authors indicated that their algorithm has a limited value when applied to larger systems.

Vouros and Papadopoulos (1998) developed a model for solving the buffer allocation problem, using a list of generic procedures that incorporated a simulation model. They studied lines of 3 and 4 stations in which MT and machine reliability were unbalanced. The processing time distributions used were exponential and Erlang. The failure rates were exponential and repair rates were exponential or Erlang- $m$ . They investigated the following patterns of combined MT and machine reliability imbalance:

- An increasing (/) MT pattern combined with balanced and bowl (V) allocation of machine failure rates (i.e. the most reliable station is located in the middle).
- A decreasing (\) MT configuration combined with a decreasing (\) arrangement of failure rates (i.e. the last station is the most reliable).

For exponential service and repair rates, it was found that in over 98% of the cases studied, an optimal buffer allocation, resulting in a maximum TR, was arrived at. However, as the number of stations increases, the efficiency of the method decreases.

Gürkan (2000) developed an efficient simulation-based optimization method for analyzing stochastic production lines having up to 50 stations, with the aim of arriving at an optimal allocation of buffers. Both MTBF and MTTR were considered as independent random variables, with deterministic, but unbalanced machine cycle times.

Papadopoulos and Vidalis (2001) developed a solution procedure to determine the optimal buffer allocation that would generate a predetermined output for MT unbalanced and unreliable serial lines with Erlang service time distribution. Lines with 3, 4, 5 and 6 stations were investigated with total buffer capacities (TBs) of 1 to 20 units. A number of MT imbalance patterns were studied. The authors stated that on average, their algorithm provided levels of accuracy on the order of 97%. Exact results were given only for a four-station line

having MTs of 3.7, 1.5, 1.1, and 3 respectively (an approximately bowl pattern). The optimal buffer arrangement for this line is an inverted bowl pattern.

Dolgui et al. (2002) studied a flow line production system, with deterministic machines having unequal production rates and subject to exponential failure and repairs rates. They presented a genetic algorithm model that is similar to that of Vouros and Papadopoulos (1998), with the exception of using exponential or Erlang, instead of deterministic machines.

A hybrid algorithm for the optimal allocation of buffers in lines having different processing rates was developed by Shi and Men (2003), who combined a Nested – Partition global search with a Tabu search method. They analysed a 9- station unreliable line with TBs of 160 and 80 units and found their hybrid method was more efficient than just using the Tabu search method.

In other studies, the focus was on the effects of breakdown, where the aim was to achieve maximum TR by optimizing mean time to repair (MTTR). A model combining simulation with queuing analysis which depicts an unreliable production system with random processing times was developed by Kouikoglou and Phillis (1994). The study goal was to determine the optimal mean time to repair (MTTR) rates that results in maximum TR. Processing times and repair rates were exponentially distributed and the model was able to accept any mean time between failures (MTBF) distribution. Results for a line having six stations with unbalanced processing times and buffers showed that output is maximized when mean processing and failure rates are equal and when faster repair rates are allocated to stations with small upstream and downstream buffer units. The authors also argued that resources (i.e. repair technicians and personnel) should be assigned in such a way that the result is a balanced line.

Nourelfath et al. (2005) took both breakdown issues and buffer placement into account in the development of a heuristic for the optimal design of a serial production line, when machine type and buffer capacity are two decision variables. They stated the problem as: given several machines (each with its own production rate, reliability and price) and several buffers (each with different capacity and cost), what is the optimal configuration of both machines and buffers in terms of line efficiency. Numerical examples for lines with 4 and 10 stations showed the efficiency of the heuristic.

Vergara and Kim (2009) considered buffer allocation in balanced and unbalanced lines. Research investigating buffer placement in unreliable lines has also been carried out to investigate the role of protective inventory in unbalanced re-entrant lines on cycle times and TR in lines under three management philosophies, traditional, just-in-time and the theory of constraints.

Shaaban (2011) studied the effects of simultaneously unbalancing MT and BC on TR, IT and ABL for 5- and 8- station *reliable* lines with different degrees of MT imbalance and low to high BCs. He found that in terms of IT, the best unbalanced pattern is an MT bowl configuration, coupled with a distribution of BC as evenly as possible. With respect to ABL, the best pattern turned out to be a monotone decreasing MT order, together with an ascending BC configuration.

Staley and Kim (2012) carried out a simulation investigation into serial production systems, subject to breakdown that is operation-dependent. Various buffer allocations and their effect

on TR were examined. A bottleneck station created by increasing MTTR was examined. The authors found that for an unbalanced, unreliable production system, a balanced allocation of buffers is best when MTTR is low, but as repair times increase, an inverted bowl allocation becomes more favorable. The maximum difference in output between the “best” and the “worst” allocation of buffers was 3%, while for reliable lines the difference turned out to be 15%.

In summary, we can see that a limited number of studies have been carried out to investigate performance of unreliable production lines with unequal processing times and uneven buffer sizes.

It can be noted that the approach taken is often with the aim of generating algorithms for the purposes of optimal or near optimal buffer allocation. There are several issues that can still be addressed following the above discussion of the literature on unbalanced, unreliable lines. Firstly, statistical analysis has not been performed to analyse whether observed changes in performance are statistically significant. Secondly, the degree of imbalance (comparing performance from lower to more extreme imbalance) has not been considered in previous studies. Thirdly, the comparative effects of different design and operating factors has not been looked into using statistical analysis to assess the relative contributions of these different elements to efficiency. Finally, there are no studies to our knowledge that observe the effects of various patterns of MT and BC allocations (and not simply total buffer capacity) on TR and ABL in unpaced lines that are suffering from machine breakdown,

We hope therefore to make three main contributions with this study:

- 1) To identify combined patterns of placing buffers (of different capacities) and workers (at different speeds) in unreliable lines which affect performance significantly, either beneficially or detrimentally, and to rank them in terms of performance.
- 2) To determine which aspects of line design; namely line length, total buffer capacity or imbalance patterns have the most impact on performance.
- 3) To provide empirical data through simulation to observe the combined effects of imbalance and unreliability on performance.

### **3. Objectives and Research Questions**

The goal of this research study is to examine the characteristics and effectiveness of unreliable lines having two concurrent sources of imbalance, caused by allowing both MTs and BCs to differ simultaneously amongst stations and buffers, whereas all CVs are held equal.

The motivation for undertaking this study is the relative lack of knowledge about the behaviour of such lines. The present paper aims to fill in many of the gaps in this area through the use of a more comprehensive and systematic investigation than hitherto attempted.

The research questions to be addressed are as follows:

- a) What are the influences of the patterns of MT&BC imbalance on the performance of the unreliable lines simulated compared to that of a balanced line?

- b) Which of the patterns simulated lead to the best performance?
- c) What are the relative contributions of imbalance patterns, imbalance degree, line length and buffer capacity to performance?
- d) What is the effect of unreliability on the performance of unbalanced MT&BC lines?

#### 4. Methodology and Experimental Design

In view of the fact that no mathematical method can currently assess the more realistic and complex serial flow lines typically reported with positively skewed operation times, computer simulation was viewed as the most suitable tool for this study

The unbalanced line behaviour was studied using a ProModel Version 7.5 coded manufacturing simulation model.

##### 4.1 Factorial Design

The most efficient and powerful of the many experimental designs is the complete factorial design. This design has therefore been chosen for the current investigation.

In order to simulate more realistic processing times, a right shifted Weibull distribution was employed. Slack (1982) reported that the unpaced service times found in real practice are more closely described by this probability function.

In the context of the particular lines being studied, the independent (exogenous) variables are:

- Line length (number of stations), **N**.
- Total amount of buffer capacity for the line, **TB**.
- Mean capacity of each buffer, **MB** (= **TB** divided by the number of buffers).
- Degree of unbalanced service time means, **DI**.
- Pattern of mean work time imbalance (**MT** pattern).
- Pattern of buffer capacity imbalance (**BC** Pattern).

And the dependent performance measures are:

- Average Buffer Level, **ABL** the amount of WIP left in the buffers after the cycle is complete for the whole line
- Throughput rate, **TR**

Evidently, the study goals are to find conditions which increase **TR** and reduce **ABL**.

##### 4.2 Statistical Tools

The following statistical techniques were used to analyse the **TR** and **ABL** data:

- Generalized Linear Model analysis (**GLM**) to identify the relative contributions of the independent variables to the dependent variable performance.
- Multiple comparisons with control using Dunnett's t-test for comparison of the performance of unbalanced lines to the balanced line control.

- Independent sample t-tests to compare performance of the two line lengths.
- Multiple pairwise comparisons (Tukey's HSD) to compare relative performance of unbalanced MT patterns, and performance of patterns at different BC levels and at different DIs.

Statistical analyses were carried out using SPSS v20.

### **4.3 Simulation Run Parameters**

All the simulation runs were started with empty buffers. In order to ensure that observations are as close to normal operating behaviour as possible, a sufficiently long warm up period is desired. The method used here is in accordance with the technique proposed by Law and Kelton (2000), i.e. to run a preliminary simulation of the system under investigation, choosing and observing one output variable, in this case WIP as they suggest. To ensure that observations are independent, minimum autocorrelation values of between -0.20 and +0.20 should be achieved (Harrell et al 2004).

A trial procedure has established that after an initial run of 20,000 minutes, acceptable autocorrelation values of between -0.163 and +0.153 were achieved, leading to the conclusion that adjacent blocks were relatively independent. In order to ensure more valid statistical data, this initial warm up period was extended to 30,000 minutes. All data collected during the first 30,000 minutes were discarded and a production run of 20,000 minutes, broken down into 50 blocks (subruns) of 400 minutes each was gathered. This resulted in mean TR and ABL values being calculated every 400 minutes and the average of these 50 mean values (the grand mean) was computed with the objective of reducing serial correlation to a negligible level.

Finally, in order to generate an identical event sequence for all the designs and highlight the contrast amongst the configurations, all the experiments used the same random number seed.

### **4.4 Failure and Repair Parameters**

The salient characteristic of unreliable lines is that the stations are subject to random mechanical failure and repair times. An exponential probability distribution with regard to both the mean time between failure (MTBF) and mean time to repair (MTTR) is considered to be most representative of what is observed in actual manufacturing systems according to an empirical study of unreliable lines by Inman (1999).

For this investigation the failure rate was set at 0.01 breakdowns per minute, with the repair rate being 0.10 repairs per minute, i.e. MTBF was 100 minutes and MTTR was 10 minutes, reflecting the rates used in the literature (Altiok and Stidham 1983, Hopp and Simon 1993). Line efficiency was thus determined to be 91% ( $MTBF\ 100 / (MTBF\ 100 + MTTR\ 10)$ ).

It was also assumed that failure and repair rates were independent, exponentially distributed random variables and as suggested by Law (2007), all downtimes were considered to be usage rather than clock based.

### **4.5 Model Assumptions**

Several relatively standard assumptions for the type of lines being studied were made. These are:



- The last station is never blocked and the first station is never starved.
- Each station is subject to failure with equal failure rates.
- Only one type of product flows in the system, with no changeovers and no defective parts being produced.
- Time to move the work units in and out of the storage buffers is negligible, hence ignored.

#### 4.6 Specific Line Design Features

While the CV value for all the stations in this investigation have equal CVs of 0.274 each, both the means and the buffer sizes were allowed to be unbalanced. The following describes the way in which this investigation was designed:

##### Constants

The CV is set at 0.274.

##### Factor Levels

- Line length: N values of 5 and 8 were specified in this study in order to look at an odd and an even number of stations.
- Total buffer capacity: TB values of 8, 24 (for N = 5), and 14, 42 (for N = 8) were selected, giving rise to MB = 2 & 6 for both N = 5 & 6. These values were selected in order that MB will not equal zero and because over a certain level of buffer space the law of diminishing returns sets in, and the increases in efficiency obtained are negligible in proportion to the extra space allocated. Note that MB refers to an average, but that the simulations in fact allocate the total buffer unevenly between stations.
- Degree of means imbalance (%): DI values of 2%, 5%, and 12% were chosen, with 2% reflecting a very slight imbalance and 12% representing a relatively high degree of imbalance.
- Means imbalance pattern: four patterns were considered:
  - A monotone decreasing order ( $\backslash$ ): the bottleneck station is positioned at the beginning of the line.
  - A monotone increasing order ( $/$ ), i.e. the station having the longest operation time (the bottleneck or constraint station) is located at the end of the line.
  - A bowl arrangement (V). Under this configuration, the constraint stations are placed at both ends of the line.
  - An inverted bowl ( $\wedge$ ), i.e. the bottleneck station is located in the middle of the line.
- Buffer allocation policies: four different policies were investigated for the total allocation of buffer size:
  - ( $/$ ): Concentrate the available buffer capacity nearer the end of the line. This

- policy displays an increasing order of buffer;
- (Λ): Concentrate the available buffer capacity nearer the middle of the line. This policy displays an inverted bowl shaped buffer size sequence;
- (∩): Concentrate the available buffer capacity nearer the beginning of the line. This policy displays a decreasing order of buffer capacity;
- No concentration of TB at one area of the line: this policy is divided into three main patterns:
  - General: No particular pattern (pattern D1).
  - Zigzag: Alternating buffer size between high and low along the line (pattern D2).
  - Bowl shape: Positioning smaller amounts of buffer towards the centre - (pattern D3).

Table 1 below shows the buffer size imbalance patterns employed:

| Line Length (N)           | 5          |              | 8                   |                      |
|---------------------------|------------|--------------|---------------------|----------------------|
| Mean Buffer Capacity (MB) | 2          | 6            | 2                   | 6                    |
| (∩)                       | 1, 1, 1, 5 | 3, 3, 3, 1 5 | 1, 1, 1, 1, 6, 2, 2 | 3, 3, 3, 3, 18, 6, 6 |
| (Λ)                       | 1, 1, 5, 1 | 3, 3, 15, 3  | 1, 1, 6, 2, 2, 1, 1 | 3, 3, 18, 6, 6, 3, 3 |
| (∩)                       | 5, 1, 1, 1 | 15, 3, 3, 3  | 6, 2, 2, 1, 1, 1, 1 | 18, 6, 6, 3, 3, 3, 3 |
| D1 (general)              | 2, 2, 3, 1 | 6, 6, 9, 3   | 2, 2, 2, 3, 3, 1, 1 | 6, 6, 6, 9, 9, 3, 3  |
| D2 (zig-zag)              | 2, 3, 2, 1 | 6, 9, 6, 3   | 2, 2, 3, 3, 2, 1, 1 | 6, 6, 9, 9, 6, 3, 3  |
| D3 (bowl)                 | 2, 1, 3, 2 | 6, 3, 9, 6   | 2, 2, 1, 1, 3, 3, 2 | 6, 6, 3, 3, 9, 9, 6  |
| Total Buffer TB           | 8          | 24           | 14                  | 42                   |

**Table 1.** Unequal buffer size allocation policies and patterns

A summary of the line design factors for the simulations carried out is given in Table 1 below:

| Performance Indicators     | Line Length (N) | Mean Buffer Capacity (MB) | Degree of Operation Time Imbalance (DI)  | MT Imbalance Patterns                      | BC Imbalance Patterns  |
|----------------------------|-----------------|---------------------------|--|--|--|
| Average Buffer Level (ABL) | 5- station line | 2, 6 units                | 2%, 5%, 12%,<br>(0% for a balanced line) | \<br>/<br>Λ<br>∩<br>---<br>(balanced line) | \<br>/<br>Λ<br>∩<br>zigzag<br>no concentration<br>---<br>(balanced line) |
| Throughput (TR)            | 8- station line |                           |  |  |  |

**Table 2.** Summary of line design features

Reminder: the coefficient of variation (CV) is fixed at 0.274 per station for all simulations.

Overall, 2 line lengths x 2 levels of BC x 3 levels of DI x 4 MT patterns x 6 BC patterns = 288 cells + 8 for the balanced line = 296 cells were simulated.

## 5. Experimental Results and Data Analysis

In the following sections the simulation results will be presented, followed by a presentation of the statistical analyses performed and an interpretation of the results, allowing us to address the research questions identified in section 3.

The data will be given only for the best and worst patterns for reasons of space. Full data are available from the authors. The results are presented in tables 3 and 4 below. The results for the balanced line are shown in the bottom line for purposes of comparison.

Significant differences with the balanced control analyzed using Dunnett's t-test are indicated with asterisks.

| <i>Best, Worst and Balanced line Throughput Results for Line Length N = 5</i> |         |         |          |          |          |          |          |
|---|---------|---------|----------|----------|----------|----------|----------|
| Mean Buffer Capacity (MB)   |         | 2       |          |          | 6        |          |          |
| % Degree of Imbalance (DI)  |         | 2       | 5        | 12       | 2        | 5        | 12       |
| MT (V)  | BC (D1) | 0.777   | 0.762    | 0.761*   | 0.850    | 0.833*   | 0.819*** |
| MT (\)  | BC (/)  | 0.756** | 0.753**  | 0.733*** | 0.822**  | 0.812**  | 0.771*** |
| Balanced Line   |         | 0.802   |          |          | 0.860    |          |          |
| <b>Throughput Results for Line Length N = 8</b>                               |         |         |          |          |          |          |          |
| MT (V)  | BC (D1) | 0.795   | 0.797    | 0.782    | 0.848    | 0.855    | 0.839    |
| MT (\)  | BC (/)  | 0.750** | 0.736*** | 0.704*** | 0.805*** | 0.812*** | 0.765*** |
| Balanced Line   |         | 0.789   |          |          | 0.870    |          |          |

**Table 3.** TR results for the best and worst performing MT & BC patterns and the balanced line. \*p< 0.05, \*\*p<0.01, \*\*\*p<0.001.

| <i>The two best and worst and Balanced line ABL Results for Line Length N = 5</i> |         |          |          |          |          |          |          |
|---|---------|----------|----------|----------|----------|----------|----------|
| Mean Buffer Capacity (MB)   |         | 2        |          |          | 6        |          |          |
| % Degree of Imbalance (DI)  |         | 2        | 5        | 12       | 2        | 5        | 12       |
| MT (\)  | BC (/)  | 0.543*** | 0.469*** | 0.291*** | 1.187*** | 1.083*** | 0.667*** |
| MT (\)  | BC (\)  | 0.725*** | 0.718*** | 0.402*** | 2.020*** | 1.113*** | 0.666*** |
| MT (/)  | BC (D2) | 1.519*** | 1.512*** | 1.691*** | 4.761*** | 4.914*** | 5.331*** |
| MT (/)  | BC (\)  | 1.341*** | 1.397*** | 1.656*** | 4.001*** | 4.851*** | 5.253*** |
| Balanced Line   |         | 0.802    |          |          | 0.860    |          |          |
| <b>Line Length N = 8</b>  |         |          |          |          |          |          |          |
| MT (\)  | BC (/)  | 0.590*** | 0.452*** | 0.381*** | 1.333*** | 1.074*** | 0.759*** |
| MT (\)  | BC (\)  | 0.887*** | 0.720*** | 0.476*** | 1.979*** | 1.297*** | 0.702*** |
| MT (/)  | BC (D2) | 1.464*** | 1.541*** | 1.660*** | 4.681*** | 5.914*** | 5.189*** |
| MT (/)  | BC (\)  | 1.348*** | 1.313*** | 1.620*** | 4.540*** | 4.633*** | 5.243*** |

|                              |       |       |
|------------------------------|-------|-------|
| Balanced Line (Best Pattern) | 0.789 | 0.870 |
|------------------------------|-------|-------|

**Table 4.** ABL results for the best and worst performing MT & BC patterns and the balanced line. \*\*\*p<0.001.

### 5.1. What is the influence of the pattern of MT&BC imbalance on the performance of the unreliable lines simulated compared to that of a balanced line?

In order to test the effects of MT & BC imbalance patterns, multiple comparisons with control using Dunnett's t-test were performed on the TR and ABL data at each level of N, MB and DI, comparing them to corresponding means obtained for the balanced control line, where MT and BC were kept equal across all stations. Those results differing significantly from the balanced line value are flagged with asterisks in the tables above.

There were only two patterns which showed slight but non-significant improvements in terms of TR for combined MT&BC unbalanced patterns over the balanced line. These were the combined MT bowl shape (V) with the fairly evenly allocated buffer (D1) at the longer line length (N=8) and lower buffer availability (MB=2) and degrees of imbalance (2% and 5%). Since none of the best unbalanced combined arrangements gave TR results that were statistically significant, we conclude that a balanced arrangement is the best as far as TR is concerned.

Overall, for TR we can see that the descending (\) MT pattern, coupled with the ascending (/) BC configuration performs considerably and significantly worse (\*\*\*p<0.001) than the balanced line at all the N, MB and DI levels investigated. Should a balanced line configuration prove to be unattainable in practice, the bowl shaped (V) MT pattern, in conjunction with the general BC pattern D1 might be used as an alternative, as no significant difference was found between the control and this particular pattern at many the factor levels considered. It seems that the effects of imbalance have a worsening effect as buffer becomes more available (higher total buffer) and as the degree of imbalance increases. Surprisingly, the effects of imbalance seem less deleterious for the longer line length (N=8) than the shorter line length (N=5). This may be because the effects of breakdown dominate over the more "minor" variability arising from imbalance as the lines get longer. This will be worth investigating in future studies.

For ABL on the other hand, the results indicate that the descending MT order, together with the ascending BC pattern A consistently shows highly significant improvements (\*\*\*p<0.001) over the balanced line, whereas the ascending MT arrangement along with the descending BC configuration C performs highly significantly worse (\*\*\*p<0.001) than the control across the board.

It should be noted that the two best patterns in terms of ABL consistently generate significantly lower ABL than the control at all N, MB and DI values simulated. Furthermore, N, MB and DI do not seem to have a consistent effect on percentage improvement in ABL over the control, with no general tendency seen in the results.

### 5.2. Which of the simultaneous MT & BC patterns simulated lead to the best TR and ABL performance?

To provide a general ranking of the patterns for overall performance, multiple pairwise comparisons (Tukey’s HSD) was performed, with the independent variable being the combined MT/BC pattern, and the dependent variables TR and ABL, respectively.

The analysis was carried out on the whole data set, with no differentiation for N, MB or DI. The results support the subjective reading of Tables 3 and 4 above, as well as the comparison carried out using Dunnett’s t-test and show the following homogeneous subgroups:

| <b>Performance Indicator</b> | <b>Subgroup 1 (Best Patterns)</b> | <b>Subgroup 2 (Medium Patterns)</b> | <b>Subgroup 3 (Worst Patterns)</b>                 |
|------------------------------|-----------------------------------|-------------------------------------|--|
|                              | MT & BC                           | MT & BC                             | MT & BC  |
| TR                           | (--)& (--) balanced               | (V) & (D1 - general)                | (\)& (/)   |
| ABL                          | (\)& (/)<br>(\)& (^)              | (V) & (/)<br>(\)& (V)<br>(^)& (/)   | (/)& (\)<br>(/)& (D2-zig-zag)<br>(/)& (D1-general) |

**Table 5.** Homogeneous subgroups for ranking of performance for different patterns of MT and BC imbalance.

The immediate conclusion to be noted from Table 5 is that the patterns of imbalance giving the best performance are different for TR and ABL. For TR, the best performing pattern is a balanced line, whereas the worst performing homogeneous subset contains the descending MT pattern, along with an ascending BC arrangement.

For ABL, the best pattern is a descending MT order, together with an ascending BC allocation. On the other hand, the worst pattern is an MT ascending arrangement, coupled with a descending BC configuration. We can also see that the worst TR pattern is actually the best configuration for ABL.

We also consider which pattern has the best effect on *combined* performance of TR and ABL. This would enable easier decision-making for line managers who have the dual objectives of increasing TR as much as possible and lowering ABL as well. In order to do this, the values of ABL and TR were normalized, and a single performance value (combining ABL and TR) was calculated, with low values corresponding to “poor” ABL and TR performance and high values corresponding to “better” performance (high TR, low ABL). Multiple pairwise comparison (Tukey’s HSD) analysis was performed, and the results were grouped and illustrated in Table 6 below. Twelve subgroups were generated, and we present the patterns in the four best and worst.

| <b>Performance Indicator</b> | <b>Best Patterns For Combined MT&amp;BC</b>  | <b>Worst Patterns For combined MT&amp;BC</b>                                |
|------------------------------|--|---|
| <b>TR + ABL (normalized)</b> | (\)& (/)<br>(\)& (^)<br>(^)& (/)<br>(V) & (/)<br>(\)&(D1-general)<br>(\)& (D2 zig-zag) | (/)& (\)<br>(/)& (D2-zig-zag)<br>(/)& (D1-general)<br>(V) & (\)<br>(^)& (\) |

**Table 6.** Homogeneous subgroups for ranking of *combined* ABL and TR performance for different patterns of MT imbalance.

We can see that the best patterns for combined performance mostly include either a descending MT pattern going from fast to slow, or an ascending buffer allocation (buffer concentrated at the end of the line). The worst combined performance often include ascending MTs or buffer concentrated at the beginning of the line. Surprisingly, pattern MT (\) & BC (/), one of the worse patterns for TR performance when considered as the sole indicator is found to be a good “medium” solution for maintaining a balance between TR and ABL performance. This can be explained by the fact that the effects of MT and BC imbalance seem to have a greater influence on ABL and a more minor effect (often not significant – see table 2) on TR. We will see in the next section that this finding is confirmed using Generalized Linear Model analysis.

### 5.3. What are the relative contributions of MT and BC patterns, DI, N and MB to performance?

Generalized Linear Model (GLM) analysis was carried out on the data in order to ascertain the relative contributions of the independent variables, namely N, MB, DI and MT & BC patterns on the dependent variables, TR and ABL. Best fit was found for a Gaussian distribution for TR and ABL. The results for TR are presented in Table 7 below. For reasons of space we have reported only on the variables of interest, namely direct contributions from MB DI, N, the MT imbalance pattern and the BC imbalance pattern and on the second order interaction between the MT and BC imbalance patterns. Full data are available from the authors.

| Throughput TR      |                 |                    | Average Buffer Level (ABL) |                 |                    |
|--------------------|-----------------|--------------------|----------------------------|-----------------|--------------------|
| Deviance = 144.230 |                 |                    | Deviance = 1,412.763       |                 |                    |
| Source (Factor)    | Wald Chi-Square | Significance Level | Source (Factor)            | Wald Chi-Square | Significance Level |
| MB                 | 2,659.796       | 0.000              | MB                         | 19,231.320      | 0.000              |
| DI                 | 511.949         | 0.000              | BC Pattern (BCP)           | 6,283.781       | 0.000              |
| BC Pattern (BCP)   | 223.184         | 0.000              | MT Pattern (MTP)           | 5,770.058       | 0.000              |
| N                  | 144.753         | 0.000              | MTP * BCP                  | 371.982         | 0.000              |
| MT Pattern (MTP)   | 125.155         | 0.000              | N                          | 158.733         | 0.000              |
| MTP * BCP          | 101.439         | 0.000              | DI                         | 157.091         | 0.000              |

**Table 7.** Selected GLM results for TR & ABL

For TR, the amount of buffer available (MB) has the strongest effect, followed by the degree of MT imbalance (DI). TR is also very highly significantly impacted by respectively, the BC pattern, line length (N) and MT imbalance pattern. The combined MT and BC pattern is the next strongest effect.

For ABL, we also see that MB has the strongest effect, but in contrast to TR performance, the patterns of BC and MT imbalance have the next strongest effects on ABL. N and DI are contributing relatively less strongly.

These results indicate the following:

- The amount of buffer available is generally the most influential factor for both TR and ABL performance.
- The patterns of imbalance are more important in ABL performance than in TR performance.
- Line length and the degree of imbalance are more influential in TR performance than in ABL performance (where the patterns of imbalance dominate).

MB is having the strongest effect on ABL, but the next strongest contribution comes from MBP, followed by MTP, the combined interaction of MTP & DI and 12 other first, second and third interactions. The impacts of N and DI on ABL are also very highly significant.

In contrast to TR, we can see that combined effects of the independent variables have generally more weight on ABL.

A more detailed analysis carried out on the individual effects of the independent variables was carried out using multiple pairwise comparisons (Tukey's HSD) giving the following general conclusions:

1) Line length has no generally significant effect on performance.

2) Increasing DI:

- causes significant deterioration in performance for TR, in particular when the imbalance gets very high (from 5% to 12%)
- enhances the effect, be it positive or negative, of the pattern of imbalance for ABL.

3) Increasing MB:

- improves performance for TR
- is less effective for higher degrees of imbalance.

In contrast, for ABL increasing buffer:

- enhances the effect, be it positive or negative, of the pattern of imbalance.

GLM analysis was repeated for the combined TR/ABL normalized performance measure to assess the joint effect of design factors for both indicators at the same time. Table 9 shows that when we consider combined TR and ABL performance, the effect of absolute buffer availability is still the strongest contributor to general performance, followed by BC and MT patterns of imbalance. This means that if both outcomes are desirable, then these three design

factors must be looked at in priority. Line length and the degree of imbalance contribute significantly, but less strongly.

| Performance Deviance           | ABL + TR (Normalized)   |                    |
|--------------------------------|-------------------------|--------------------|
|                                | 620.877                 |                    |
| Source (Factor)                | Wald Chi-Square         | Significance Level |
| <b>MB</b>                      | 8995.923 <sup>***</sup> | 0.000              |
| <b>MT Pattern</b>              | 2691.079 <sup>***</sup> | 0.000              |
| <b>MT Pattern * DI</b>         | 2311.821 <sup>***</sup> | 0.000              |
| <b>BC Pattern</b>              | 2300.060 <sup>***</sup> | 0.000              |
| <b>MT Pattern * MB</b>         | 2232.057 <sup>***</sup> | 0.000              |
| <b>MB * BC Pattern</b>         | 1143.508 <sup>***</sup> | 0.000              |
| <b>MT Pattern * BC Pattern</b> | 391.764 <sup>***</sup>  | 0.000              |
| <b>N * BC Pattern</b>          | 192.669 <sup>***</sup>  | 0.000              |
| <b>DI * BC Pattern</b>         | 148.921 <sup>***</sup>  | 0.000              |
| <b>MB * DI</b>                 | 117.104 <sup>***</sup>  | 0.000              |
| <b>N</b>                       | 92.404 <sup>***</sup>   | 0.000              |
| <b>DI</b>                      | 79.843 <sup>***</sup>   | 0.000              |
| <b>MB * N</b>                  | 46.744 <sup>***</sup>   | 0.000              |
| <b>N * DI</b>                  | 12.723 <sup>**</sup>    | 0.002              |
| <b>MT Pattern * N</b>          | 11.412 <sup>**</sup>    | 0.010              |

**Table 9.** GLM Analysis for combined ABL and TR performance. <sup>\*\*\*</sup> p<0.001; <sup>\*\*</sup> p≤0.01

#### 5.4. What is the effect of unreliability on the ABL performance of unbalanced MT&BC lines?

A comparison of some of the results presented here with those published by Shaaban (2011) on reliable lines with the same design characteristics can be effected by comparing the relative changes in performance in ABL. He published the percentage improvements in ABL performance for the best combined MT&BC unbalanced pattern, and we display in Table 9 the data for the equivalent unreliable lines, where we have corresponding results. The numbers in bold show where the unreliable line is performing better than the reliable counterpart.

| Pattern MT (\) & BC (/): Percentage Reductions in ABL |           |          |              |          |              |
|---|-----------|----------|--------------|----------|--------------|
|   |           | N = 5    |              | N = 8    |              |
| MB  | % DI      | Reliable | Unreliable   | Reliable | Unreliable   |
| <b>2</b>  | <b>2</b>  | 54.79    | 44.37        | 51.96    | 41.06        |
|   | <b>5</b>  | 65.83    | 51.95        | 60.31    | 54.85        |
|   | <b>12</b> | 79.14    | 70.18        | 74.64    | 61.94        |
| <b>6</b>  | <b>2</b>  | 51.22    | <b>63.65</b> | 37.37    | <b>58.36</b> |
|   | <b>5</b>  | 77.34    | 66.83        | 63.80    | <b>66.45</b> |



|  |           |       |       |       |       |
|--|-----------|-------|-------|-------|-------|
|  | <b>12</b> | 89.58 | 79.57 | 84.58 | 76.29 |
|--|-----------|-------|-------|-------|-------|

**Table 9.** Comparison of unreliable and reliable lines. Percentage savings in ABL for best pattern MT (\) + BC A (/), compared to the balanced control. \*source: Shaaban (2011)

When the ABL results in Table 9 are examined, it can be noted that in most instances, the reliable line gave higher savings in ABL performance over the balanced line, as compared to the unreliable line. It is to be expected that lines suffering from breakdown will perform less well than their reliable counterparts, since we are adding even more fluctuation into the line, and intuitively one would expect an increase in in work-in-process when lines suffer from downtime. However, in a few cases (in bold in table 9), we can see that this is not so when available buffer space is higher (MB = 6) and when MT imbalance is not extreme (DI = 2% or 5%). It can also be observed that for both reliable and unreliable lines the ABL improvements were accentuated as imbalance grew from % DI = 2 to 12 and as the line shortened. Furthermore, for unreliable lines only, the superiority in ABL grew as MB was increased from 2 to 6 units, indicating that high buffer space availability is of use in compensating for breakdown effects and less useful in reliable lines beyond a certain capacity.

## 6. Summary

A number of MT and BC unbalancing policies and configurations were analysed in this study. We can draw a number of conclusions from the findings presented above. Firstly, the patterns of imbalance of mean operation times and mean buffer capacity along a production line that is subject to machine breakdown has a clear influence on performance.

Secondly, a balanced configuration is the best when throughput is the performance measure of interest. This generally agrees with the results of Staley and Kim (2012) that for an unbalanced, unreliable production system, a balanced allocation of buffers is best when MTTR is low. In our investigation a relatively low MTTR rate of 10 minutes was used.

In contrast, when we observe the results in terms of average buffer level performance, the pattern giving rise to the lowest ABL is a descending MT order (\), in conjunction with an ascending (/) BC configuration This agrees with Shaaban (2011) finding for reliable line performance.

Thirdly, it should be noted that for the pattern giving the best ABL results, consistent improvements over the balanced line are obtained for practically all the N, BC, and DI values explored.

In addition to the effects of patterns of MT and BC imbalance, a number of observations can be made about the other design factors, namely buffer capacity, degree of MT imbalance and line length on performance. In general, MB was seen to have the biggest influence on performance of ABL and TR separately, the degree of imbalance has more effect on MT than ABL, and the patterns of imbalance are generally more influential for ABL performance than for throughput.

Of interest to practitioners, our GLM analysis shown that the way buffer is allocated in unreliable lines has a bigger influence on both MT and ABL performance than the positioning of workstations with differing mean times (MT imbalance pattern). However, both are of importance, particularly for ABL performance.

A more detailed analysis shows that as buffer capacity increases, throughput rises. The general effect of higher buffer availability on ABL is to enhance the effect of imbalance be that positive or negative, with poorly imbalanced patterns showing declining performance relative to the balanced line with rising buffer, and beneficial patterns of imbalance continuing to improve ABL performance compared to the balanced control, as more buffer is added.

On examination of the influence of the degree of mean time imbalance, we can see that when DI increases, throughput does not change much, except for when imbalance reaches higher levels of  $DI\% = 12$ , whereas ABL falls, i.e. more extreme imbalance actually can have a positive effect in terms of ABL performance if the line is unbalanced in the correct fashion. There have been no previous studies to our knowledge which explicitly investigate the effects of degree of imbalance, but the data reported by Das et al (2010) seem to indicate that percentage working time along an unbalanced line increases as the degree of imbalance grows, and then deteriorates on further imbalance which lends support to the findings for TR presented here.

Finally, in terms of improvement in performance, the greatest % improvement in ABL over the unreliable balanced line counterpart is: ABL 79.6% (very highly significant).

When data are compared to the ABL results reported by Shaaban (2011) for a corresponding *reliable* MT&BC study, we find similarity in the best unbalanced pattern found, as well as in the operating behaviour of both unbalanced reliable and unbalanced unreliable production flow lines. The observation that in some cases the unreliable lines are performing better in terms of ABL than their reliable counterparts when compared to the balanced line, is an issue that needs to be explored further in future studies.

## **7. Discussion and Conclusions**

In the introduction we noted the fact that observations in real production systems, particularly manual or man-machine operated lines show that balance is an ideal which many strive for but which is seldom reached. This being the case, some scholars have turned their attention to how best to deal with these lines which are subject to variability arising from human differences and unexpected events such as breakdown.

The aim of the simulations carried out here was to model a limited number of such systems to provide insights into what kind of factors might improve performance and attenuate those conditions which are detrimental to performance such as breakdown. As with all experimental methods, the aim is to determine cause and effect in controlled conditions, and not to suggest general rules which other approaches such as algorithm generation and modelling based on extensive field studies might provide.

We find that superior performance to that achieved by a balanced line in terms of average buffer level is attainable. When ABL is considered, the savings obtained are very significant (79.57% for the best case). If lean buffering is the aim, then an unbalanced line seems to be

positively advantageous especially since the improvement in average buffer level only requires appropriately assigning line operators to the same stations, which does not entail any extra expenditure on capital or other resources.

Managers, however, would have to make a choice if they wished to redesign their lines through MT and BC allocation. Our findings indicate that none of the patterns considered simultaneously achieved low TR and low ABL levels. The relative costs of inventory and lost production need to be taken into account when decisions are made. This being the case, a line manager will opt for a line design where the greatest advantage can be gained. This in turn will depend greatly on the sector, the nature of the product and the relative costs of labour and inventory. For fast-moving consumer goods, output will be of the essence, and so here those lines close to balance will be preferred. Similarly in knowledge-intensive industries such as medical, chemicals or pharmaceuticals where labour costs are necessarily high, idle time, inversely proportional to throughput needs to be decreased. In industries where storage space is at a premium, or where perishable goods are involved, managers could opt for unbalanced allocations of MT and BC as observed here in order to keep costs and risks down.

Following on from this research, several possibilities are open. We used fixed breakdown and repair rates here for reasons of experimental control, but it would be of interest to vary breakdown and repair rates at different stations to mirror real systems even more closely. Other possibilities could be to use costs and profits as performance indicators in these unbalanced lines, especially in view of the incompatibility of increasing throughput while also keeping inventory low.

## References

- Altiok, T., & Stidham, S. (1983). The allocation of inter-stage buffer capacities in production lines. *IIE Transactions*, Vol. 15, No. 4, 292-299.
- Dolgui, A., Ereemeev, A., Kolokolov, A., & Sigaev, V. (2002). A genetic algorithm for the allocation of buffer storage capacities in a production line with unreliable machines. *Journal of Mathematical Modelling and Algorithms*, Vol. 1, 89-104.
- Gershwin, S. B., & Schor, J. E. (2000). Efficient algorithms for buffer space allocation. *Annals of Operations Research*, Vol. 93, 117-144.
- Gürkan, G. (2000). Simulation optimization of buffer allocation in production lines with unreliable machines. *Annals of Operations Research*, Vol. 93, Nos. 1-4, 177-216.
- Harrell, C., Ghosh, B. K., & Bowden, R. O. (2004). *Simulation Using ProModel*. (3<sup>rd</sup> ed.) New York: McGraw – Hill, (Chapter 7).
- Heavey, C., Papadopoulos, H. T., & Browne, J. (1993). The throughput rate of multistation unreliable production lines. *European Journal of Operational Research*, Vol. 68, 69-89.
- Ho, Y. C., Eyler, M. A., & Chien, T. T. (1979). A gradient technique for general buffer storage design in a production line. *International Journal of Production Research*, Vol. 17, No. 6, 557-580.

- Hopp, W. J., & Simon, J. T. (1993). Estimating throughput in an unbalanced assembly like flow system. *International Journal of Production Research*, Vol. 31, No. 4, 851 -868.
- Inman, R. R. (1999). Empirical evaluation of exponential and independence assumptions in queuing models of manufacturing systems. *Production and Operations Management*, Vol. 8, No. 4, 409 - 432.
- Kouikoglou, V. S., & Phillis, Y. A. (1994). Discrete event modelling and optimization of unreliable production lines with random rates. *IEEE Transactions on Robotics and Automation*, Vol. 10, No. 2, 153-159.
- Law, A. M. (2007). *Simulation modeling and analysis*. (4<sup>th</sup> ed.) New York: Irwin / McGraw Hill, (Chapter 5).
- Law, A. M., Kelton, W. D. (2000). *Simulation modeling and analysis* (3<sup>rd</sup> ed.) New York: Irwin / McGraw Hill, (Chapter 11).
- Nahas, N., Ait-Kadi, D., & Nourelfath, M. (2006). A new approach for buffer allocation in unreliable production lines. *International Journal of Production Economics*, Vol. 103, 873-881.
- Nourelfath, M., Nahas, N., & Ait-Kadi, D. (2005). Optimal design of series production lines with unreliable machines and finite buffers. *Journal of Quality Maintenance Engineering*, Vol. 11, No. 2, 121-138.
- Papadopoulos, H. T., & Vidalis, M. I. (2001). A heuristic algorithm for the buffer allocation in unreliable unbalanced production lines. *Computers & Industrial Engineering*, Vol. 41, 261-277.
- Shaaban, S. (2011). Unpaced production lines with jointly unbalanced operation time means and buffer capacities. *International Journal of Manufacturing Technology and Management*, Vol. 23, Nos. 1/2, 54 - 68.
- Shaaban, S., & Hudson, S. (2009). The performance of unpaced serial production lines with unequal coefficients of variation of processing times. *International Journal of Computer Applications in Technology*, Vol. 34, No.2, 122-128.
- Shaaban, S. & McNamara, T. (2009). Improving the efficiency of unpaced production lines by unbalancing service time means. *International Journal of Operational Research*, Vol. 4, No. 3, 346-361.
- Slack, N. (1982). Work time distributions in production system modelling. Research paper, *Oxford Centre for Management Studies*.
- Shi, L. & Men, S. (2003). Optimal buffer allocation in production lines. *IIE Transactions*, Vol. 35, 1-10.
- Staley, D. R., & Kim, D. S. (2012). Experimental results for the allocation of buffers in closed serial production lines. *International Journal of Production Economics*, Vol. 137, 284-291.

Tempelmeier, H. (2003). Practical considerations in the optimization of flow production systems. *International Journal of Production Research*, Vol.41, No.1, 149-170.

Vergara, H.A., & Kim, D.S. (2009). A new method for the placement of buffers in serial production lines. *International Journal of Production Research*, Vol. 47, 4437-4456.

Vouros, G. A., & Papadopoulos, H. T. (1998). Buffer allocation in unreliable production lines using a knowledge based system. *Computers and Operations Research*, Vol. 25, No. 12, 1055-1067.